FLUID MECHANICS

A POINT EXPLOSION IN AN INHOMOGENEOUS ATMOSPHERE

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It is known that the problem concerning a point explosion in a medium with a constant adiabatic exponent γ has an analytic solution [1] for that stage of the phenomenon, during which the energy in the medium before the explosion can be neglected in comparison with the energy involved in the explosion wave. A relatively simple solution can be obtained, because the problem is self-simulating, and does not involve any characteristic parameters such as the linear dimension, the velocity, or the time. If a density gradient exists, even if in only one direction, the problem is no longer self-simulating, and an exact solution cannot be obtained. If the gradients are small, then the problem can be successfully attacked by using the method of small variations [2,3].

If the explosion takes place in a very rarefied atmosphere, then a strong shock wave, with the maximum compression, is propagated to such a great distance that the density may vary by many times its original value at the point of the explosion. In such a case, linearization is not permissible. The use of numerical methods when there are three variables is found to be very laborious, even when an electronic computer is used.

A partly qualitative approach can be proposed, which is based on an essential property of a point centrally symmetric solution. This means, in this type of solution, the energy is distributed almost uniformly in the whole volume of the explosion wave, and only close to the front does the energy density exceed by two or three times the mean value through the volume. It is in this region that all the mass is concentrated.

It is natural to assume that this same phenomenon takes place in an explosive wave in an inhomogeneous atmosphere. Actually, if the pressure inside the wave is constant in space (the pressure proportional to the energy density), and the mass density is zero, then the hydrodynamic equations are satisfied in the main part of the volume in a trivial way. Then, in order to describe the propagation of the wave, the conditions must be fulfilled at the front itself.

If the equation of the wave front is $f(r, z, t) = 0$, then the normal velocity component of the front D_n will be determined by the known equality

$$D_n = -\frac{\partial f}{\partial t} \Big/ |\nabla f| = \sqrt{\frac{p}{\rho^2\left(\frac{1}{\rho} - \frac{1}{\rho'}\right)}}. \tag{1}$$

Here, as is usual in the case of powerful explosions, the initial pressure is neglected in comparison with the pressure p in the front of the wave. In this approximation, the ratio of the density behind the front to the density ahead of the front, is constant,

$$\frac{\rho'}{\rho} = \frac{\gamma+1}{\gamma-1}. \tag{2}$$

The pressure is expressed in terms of the energy density ϵ as

$$p = (\gamma - 1)\,\varepsilon = (\gamma - 1)\,\lambda \frac{E}{V}, \tag{3}$$

where E is the total energy of the explosion, V is the volume occupied by the explosion wave, and $\lambda = \lambda(\gamma)$ is a coefficient giving the ratio of the energy density at the front to the mean energy density through the volume [1]. The assumption that λ is constant over the surface forms the basis of the method proposed here.

We will assume that the equation of the wave front in cylindrical coordinates has been solved for the radius, and $r = r(z, t)$. Then the total volume included inside the front will be

$$V(t) = \pi \int_{z_1}^{z_2} r^2(z, t)\, dz, \tag{4}$$

where $r(z_1, t) = r(z_2, t) = 0$. If (2), (3), and (4) are substituted in (1), and the density is expressed according to the barometric formula, then we obtain, for the function r, the partial differential equation

$$\left(\frac{\partial r}{\partial y}\right)^2 - e^{-z/z_0}\left[\left(\frac{\partial r}{\partial z}\right)^2 + 1\right] = 0. \tag{5}$$

Here z_0 is the equivalent thickness of the atmosphere, y is an auxiliary variable defined by the equality

$$y = \int_0^t \frac{dt}{\sqrt{V}} \sqrt{\frac{\lambda E(\gamma^2 - 1)}{2\rho_0}}; \tag{6}$$

and ρ_0 is the original density of the air at the point of explosion ($z = 0$).

Equation (5) is solved by separating the variables, and

$$r = \xi y + \int_0^z dz \sqrt{\xi^2 e^{-z/z_0} - 1}; \tag{7}$$

$$\frac{\partial r}{\partial \xi} = y + \int_0^z dz \frac{\xi e^{-z/z_0}}{\sqrt{\xi^2 e^{-z/z_0} - 1}} = F(\xi). \tag{8}$$

For small t or y, the wave must be spherical. In this case, it is sufficient to set the function $F(\xi)$ equal to zero. If ξ is eliminated from (7) and (8), we obtain

$$r = 2z_0 \arccos\left[\frac{1}{2} e^{z/2z_0}\left(1 - x^2 + e^{-z/z_0}\right)\right]; \tag{9}$$

where $x = y/2z_0$.

From this we can obtain the position of the highest and lowest points z_1 and z_2 of the wave

$$e^{-z_{1,2}/2z_0} = 1 \mp x, \tag{10}$$

and, also, the position and magnitude of the maximum radius

$$e^{-z_m/z_0} = 1 - x^2, \qquad r_m = 2z_0 \arcsin x. \tag{11}$$

The maximum possible radius of the wave is, therefore, πz_0. Here $x = 1$, so that the upper limit of the wave recedes to infinity. But this occurs at a finite time τ, which is found from (6) to be

$$\tau = \sqrt{\frac{8\pi z_0^5 \rho_0}{\lambda E(\gamma^2 - 1)}} \int_0^1 \sqrt{\Omega(x)}\, dx, \tag{12}$$

where

$$\Omega(x) = \int_{-2\ln(1+x)}^{-2\ln(1-x)} du \operatorname{arc\,cos}^2\left[\frac{1}{2} e^{u/2}(1 - x^2 + e^{-u})\right]. \tag{13}$$

The time at which the wave tends to infinity appears to be finite because the velocity of the front, according to (1), tends to infinity for $z \to \infty$. Figure 1 gives the curves relating $\underline{x}$ to $\underline{t}$ and to $[\Omega(x)]^{\frac{1}{2}}$. In Fig. 2 are drawn to scale the calculated sections of the wave by a vertical plane passing through the point of the explosion, for certain instants of time.

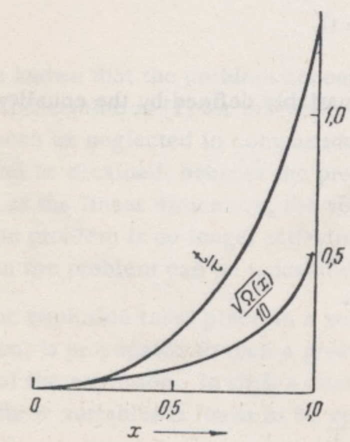

Fig. 1.

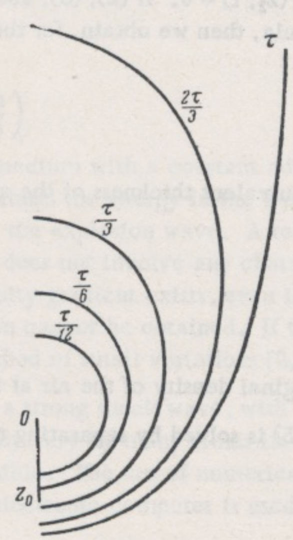

Fig. 2.

The solution naturally loses all reality at some time before z_1 becomes infinite. The following conclusion can nevertheless be drawn: no matter how great the energy of the explosion, a strong shock wave, according to the results obtained here, cannot penetrate downwards further than 1.38 z_0, or approximately 11 km. For any penetration below this altitude, the shock wave will be rapidly weakened, because of the rarefaction waves travelling upwards from it, into the region that is open, to the empty space above, where these waves will be lost. The propagation of the wave in the motionless air brings to mind the phenomenon of a short impulse in a substance bounded by a vacuum, which has been studied by Ya. B. Zel'dovich [4].

LITERATURE CITED

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