A theory of energization of solar wind electrons by the Earth's bow shock

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Received 03/01/84, accepted 16/03/84.

ABSTRACT. A theory of the reflection and energization of some of the incoming solar wind electrons by the Earth's bow shock is presented. The theory is based on essentially adiabatic mirror reflection, in the appropriate reference frame, of incident electrons by the rise in magnetic field magnitude which takes place in the shock transition region. The average energy per backstreaming electron scales for sufficiently large θ_{Bn} as $\sim 4/\cos^2 \theta_{Bn}(1/2\, mV_0^2)$, where θ_{Bn} is the angle between upstream magnetic field and shock normal, m is the electron mass and V_0 the solar wind bulk velocity. This energy can be very large as the shock becomes close to perpendicular. Expressions for the density, flux, energy per charge, temperature anisotropy and average pitch-angle of the reflected energetic electrons are derived. The comparison between the theory and observations of energetic electrons in the range $\sim$ 50 eV-50 keV in the upstream region of the Earth's bow shock shows satisfactory agreement. It is conjectured that the present theory may be applied as well to interplanetary shocks, theta-pinch laboratory experiments and solar type II bursts radiation emission containing herringbone structure.

Key words : Earth bow shock, collisionless shocks, electron acceleration.

Annales Geophysicae, 1984, **2**, 4, 449-456.

I. INTRODUCTION

It is now well established experimentally that energetic electrons in the energy range 50 eV-50 keV originate rather steadily at the Earth's bow shock and propagate upstream in the sunward direction along the interplanetary magnetic field lines (Fan *et al.*, 1964 ; Anderson, 1968 ; Ogilvie *et al.*, 1971 ; Feldman *et al.*, 1973 ; Scudder *et al.*, 1973 ; Reasoner, 1975 ; Parks *et al.*, 1978 ; Anderson *et al.*, 1979 ; Anderson, 1981 ; Parks *et al.*, 1981 ; Feldman *et al.*, 1982, 1983). The most energetic electrons (energies as high as 50 keV) are detected at the edge of the foreshock region (i.e., the upstream region magnetically connected to the Earth's bow shock) and come primarily from a small region near the curve of tangency of the interplanetary magnetic field to the bow shock surface, where the shock is nearly perpendicular ; lower energy electrons (down to 10 eV) are detected deeper in the foreshock and are released from a much broader region of the bow shock (Anderson *et al.*, 1979 ; Feldman *et al.*, 1983). The energetic electrons have also been shown to be closely associated with the presence of predominantly electrostatic electron plasma waves in the foreshock region (Scarf *et al.*, 1971 ; Filbert and Kellogg, 1979 ; Anderson *et al.*, 1981 ; Etcheto and Faucheux, 1984 ; Lacombe *et al.*, 1984), and to act as a source of electromagnetic waves at twice the electron plasma frequency (Hoang *et al.*, 1981). These various waves carry however little energy in comparison with the energetic electrons (Fredricks *et al.*, 1971), which is consistent with the fact that these energetic electrons are detected far upstream by lunar based instruments (Reasoner, 1975) and even farther by the ISEE 3 spacecraft (Anderson, 1981 ; Feldman *et al.*, 1983), with essentially the same characteristics as near, but upstream of, the Earth's bow shock.

Energetic ($\sim$ 2 keV) electrons have also recently been detected upstream of interplanetary shocks (Potter, 1981), which as compared to the Earth's bow shock are generally characterized by a lower Mach number and similar ratio of thermal to magnetic energy.

While the observations have reached a rather mature state, the issue of the physical understanding of the acceleration of solar wind electrons by shocks is still unsettled. It is generally assumed that wave-particle interactions within the shock produce sufficient electron acceleration. Thus, in a model elaborated by Smith (1971) and applied to solar type II bursts phenomena, the electrons are heated by ion acoustic turbulence behind the shock front, which allows fast electrons from the Maxwellian tail to escape to the upstream region. It has also been suggested that instabilities driven by the ions reflected by supercritical, nearly perpendicular shock waves (Wu *et al.*, 1984) can lead in their nonlinear phase to proper electron acceleration (Papadopoulos, 1981 ; Tanaka and Papadopoulos, 1983). The approach adopted here is of different nature, and is based on the following general framework :

(i) the jump in intensity of the macroscopic magnetic field at the shock transition acts as a magnetic mirror and combines with a jump of the electric potential to reflect adiabatically part of the incident solar wind electron distribution (Feldman *et al.*, 1983).

(ii) the energy of an electron is being conserved before and after reflection only in a particular reference frame where the motional electric field is zero. This frame moves at a very large relative velocity with respect to the observer's reference frame for nearly perpendicular shock geometries ; thus important energization results when coming back to the observer's frame.

Such a mechanism is similar to that investigated earlier in the context of the acceleration of cosmic rays by shock waves when scattering is neglected (see the reviews by Toptyghin (1980) and Axford (1981)). However, the analysis of scatter-free shock acceleration of cosmic rays is generally concerned with already energetic incident particles and thus relies on the assumption that the shock transition thickness can be considered as small compared to the incident particle gyroradius. The opposite regime holds instead in the present analysis (gyroradius $\ll$ shock thickness ; adiabatic reflection), as we focus on the reflection and energization of essentially thermal or moderately superthermal incoming electrons. However, both regimes lead to the important result that the average energy per backstreaming particle scales with θ_{Bn} (the angle between interplanetary magnetic field and local shock normal) as $1/\cos^2 \theta_{Bn}$ (for sufficiently large θ_{Bn}), and thus can be very large as the shock becomes close to perpendicular. Such a scaling, expected for any mechanism which relies on the conservation of energy through the reflection in a frame where the motional electric field is zero, has been obtained and used in particular by Sonnerup (1969) (energization of thermal ions reflected at the Earth's bow shock), Potter (1981) (electron energization at interplanetary shocks) and recently by Holman and Pesses (1983) (electron energization at shocks in the solar corona and solar type II radio emission).

Previous work, however, was limited to the discussion of the behaviour of one given particle in the reflection process. This paper is devoted to the derivation of the distribution function and various macroscopic moments (density, bulk velocity, energy, temperatures parallel and perpendicular to the magnetic field, average pitch-angle) of the reflected electrons when emitted by the shock, as a function of the shock parameters and geometry. The comparison, made in Section III, between the results and available observations of energetic electrons in the upstream region of the Earth's bow shock shows satisfactory agreement.

The present work was nearly completed when we became aware of the existence of another work on the same subject based on very similar concepts (C. S. Wu, submitted to *J. Geophys. Res.*, Oct. 1983). Main differences are that the electric potential in the shock transition is neglected, and a bi-Maxwellian is taken to approximate the incoming electron distribution in Wu's approach, whereas the electric potential is kept and a single Maxwellian represents the solar wind electrons in the present work.

II. THE MODEL

a) Definitions and main assumptions

We consider the interaction of an incoming solar wind electron plasma with the bow shock surface. We assume that at each point of the bow shock surface the reflection process takes place locally, i.e., that over the distance covered by an electron from the point where it enters the shock up to the point where it emerges upstream the shock parameters (upstream Mach number and temperature of each particle species, angle between magnetic field and shock normal, etc.) have not varied significantly. We also assume that during the transit time of an incoming electron within the shock transition, the shock can be treated as planar, one-dimensional (along the shock normal $\mathbf{n}$), and stationary.

Two different reference frames are considered. The first of these is fixed with respect to the bow shock surface and is denoted « observer's frame » in the following, as in general the observing spacecraft are nearly motionless relative to the bow shock. In this reference frame, and at a given location on the bow shock surface, can be defined the incoming (solar wind) plasma bulk velocity $\mathbf{V}_0$, the local shock normal $\mathbf{n}$ and the unit vector $\mathbf{b}$ along the interplanetary magnetic field $\mathbf{B}_0$; these vectors form in general a three-dimensional structure (figure 1). The angles between $\mathbf{b}$ and $\mathbf{n}$, $\mathbf{V}_0$ and $\mathbf{n}$, and $\mathbf{b}$ and $\mathbf{V}_0$ are denoted θ_{Bn}, θ_{Vn} and θ_{BV} respectively. The sense of $\mathbf{n}$ and $\mathbf{b}$ is chosen so that both $\mathbf{n}$ and $\mathbf{b}$ point to the downstream half space (as $\mathbf{V}_0$ does), which ensures that $\cos \theta_{Vn}$ and $\cos \theta_{Bn}$ be positive. The x axis is defined to be along $\mathbf{n}$, to point downstream and to have an origin $x = 0$ located upstream just at the beginning of the shock transition. The second reference frame of interest, the so-called de Hoffman and Teller frame (hereafter HT frame), is defined such that the solar wind bulk velocity as seen in this frame is aligned with the interplanetary magnetic field, which has the effect of removing the motional electric field $-\mathbf{V} \times \mathbf{B}$. The HT frame moves with respect to the observer's frame with a velocity $\mathbf{V}_{HT}$ contained in the shock plane given by (Landau and Lifshitz, 1957)

$$\mathbf{V}_{HT} = \frac{\mathbf{n} \times (\mathbf{V}_0 \times \mathbf{b})}{\mathbf{n} \cdot \mathbf{b}}. \quad (1)$$

From eq. (1) it is clear that a singularity arises for θ_{Bn} equal to 90°. In what follows we shall restrict ourselves to shock geometries such that $\theta_{Bn} \leqslant 89.5°$, for which under solar wind conditions the velocity $\mathbf{V}_{HT}$ is subrelativistic and the reference frames transformations may be treated within the context of classical mechanics.

We define here some notations. All quantities evaluated in the HT frame are denoted with a prime, while the corresponding quantities in the observer's frame are written without prime. Also, unless otherwise specified, all quantities in any frame written without argument (x) are understood to be evaluated in the upstream region $(x = 0)$. We define moreover the velocities $\mathbf{v}_\parallel$ and $\mathbf{v}_\perp$ (or $\mathbf{v}'_\parallel$ and $\mathbf{v}'_\perp$) parallel and perpendicular to $\mathbf{B}_0$ of a particle with velocity $\mathbf{v}$ (or $\mathbf{v}'$) by the relations

$$\mathbf{v} = v_\parallel \, \mathbf{b} + c \frac{\mathbf{E} \times \mathbf{B}_0}{B_0^2} + \mathbf{v}_\perp \tag{2}$$

$$\mathbf{v}' = v'_\parallel \, \mathbf{b} + \mathbf{v}'_\perp \tag{3}$$

where $\mathbf{E}$ is the upstream electric field in the observer's frame (recall $\mathbf{E}' = 0$). It is then easily shown using standard fields and velocities transformations between reference frames and using the definitions (2) and (3) that

$$v'_\parallel = v_\parallel - \mathbf{V}_{\mathrm{HT}} \cdot \mathbf{b} \tag{4}$$

$$\mathbf{v}'_\perp = \mathbf{v}_\perp \,. \tag{5}$$

It follows from equations (1)-(5), or directly by substracting $\mathbf{V}_{\mathrm{HT}}$ from $\mathbf{V}_0$, that the upstream bulk velocity in the HT frame is related to V_0 by

$$\mathbf{V}'_0 = V'_0 \, \mathbf{b} = V_0 \frac{\cos \theta_{Vn}}{\cos \theta_{Bn}} \mathbf{b} \,. \tag{6}$$

Note that V'_0 can become much larger than V_0 when the shock becomes nearly perpendicular (fig. 1). Note also that a positive (negative) sign of $v'_\parallel$ (but not $v_\parallel$) means that the electron is moving towards (away from) the shock surface.

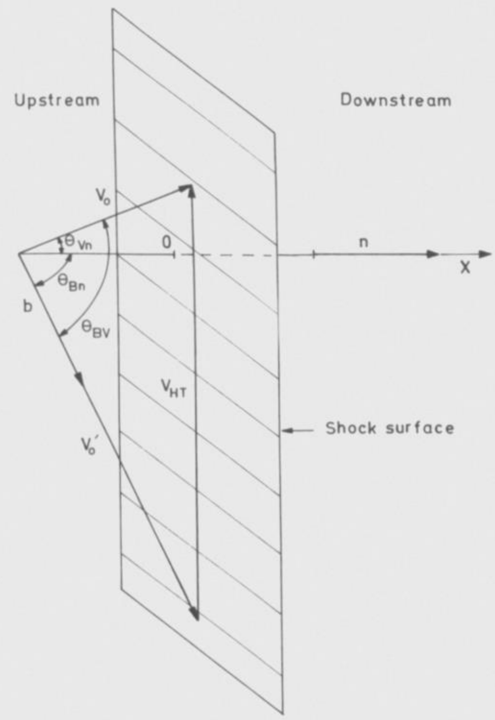

Figure 1.
Shock geometry. Note that for large angles θ_{Bn}, V'_0 is much larger than V_0.

b) Reflection mechanism

In the HT frame the electromagnetic field structure of the shock simplifies considerably due to the fact that the relation $\mathbf{E}'(x) \times \mathbf{n} = 0$ holds within the entire shock transition region. Thus the electric field in the shock can simply be represented by an electric potential $\phi'(x)$.

Moreover the dynamics of an electron in the shock can be described in the HT frame using standard adiabatic theory (e.g., Morozov and Soloviev, 1966). A necessary condition for adiabatic theory to be valid is that the ratio of an electron gyroradius v_{th}/Ω_e be small compared to a characteristic length L of change of the ambient electromagnetic field (v_{th} : electron thermal speed; Ω_e : electron gyrofrequency). We take L to be the width of the shock transition region, defined here as the spatial length along $\mathbf{n}$ over which the magnitude of the magnetic field $B(x)$ and the potential $\phi'(x)$ rise from their upstream to their maximum value. It is observationally well-known that L is of the order of an ion inertial length c/ω_i for quasi-perpendicular shocks, where ω_i is the upstream ion plasma frequency (Russell and Greenstadt, 1979; Russell *et al.*, 1982). Thus, denoting by m and M the electron and ion mass, and by β_e the ratio of electron thermal to magnetic pressures, the ratio $(v_{\mathrm{th}}/\Omega_e)(1/L)$ is found to be of the order of $(m/M)^{1/2} \beta_e^{1/2}$, which under solar wind conditions is indeed a small number in comparison with 1. The relevant energy equation of one electron moving through the shock transition in the HT frame may then be written (Morozov and Soloviev, 1966):

$$\frac{1}{2} m v'^2_\parallel(x) + \mu B(x) - e\phi'(x) = \text{Constant} \tag{7}$$

where e is the elementary charge ($e \geqslant 0$) and μ is the magnetic moment

$$\mu = \frac{1}{2} m \frac{v'^2_\perp(x)}{B(x)} = \text{Constant} \,. \tag{8}$$

The subscripts $\parallel$ and $\perp$ in equations (7)-(8) refer to directions parallel and perpendicular to the *local* magnetic field $\mathbf{B}(x)$, not $\mathbf{B}_0$. Note that the displacements of the electron perpendicular to $\mathbf{n}$ do not lead to any work exchange, since $\mathbf{E}' \times \mathbf{n} = 0$. In any other reference frame than the HT frame would appear in equation (7) additional terms such as $e \int E_y \, dy$ that cannot possibly be evaluated since the transverse displacements of the electron within the shock transition region do not obey any simple law. Note also that in writing equation (7) we have assumed that the drift approximation of the adiabatic theory holds, which is valid when the drift velocity $c \,|\, \mathbf{E}' \times \mathbf{B} \,|/B^2$ is small in comparison with v_{th} (Morozov and Soloviev, 1966). This is justified, since $E' \sim \phi'/L \sim T_e/eL$ (Axford, 1981; Goodrich and Scudder, 1984; Schwartz and Feldman, 1984), and

$$c \,|\, \mathbf{E}' \times \mathbf{B} \,|/B^2 \, v_{\mathrm{th}} \sim (m/M)^{1/2} \beta_e^{1/2} \ll 1$$

under solar wind conditions.

It is convenient (Feldman *et al.*, 1983) to introduce a pseudo-potential $\Psi(x)$ and to rewrite eq. (7) as

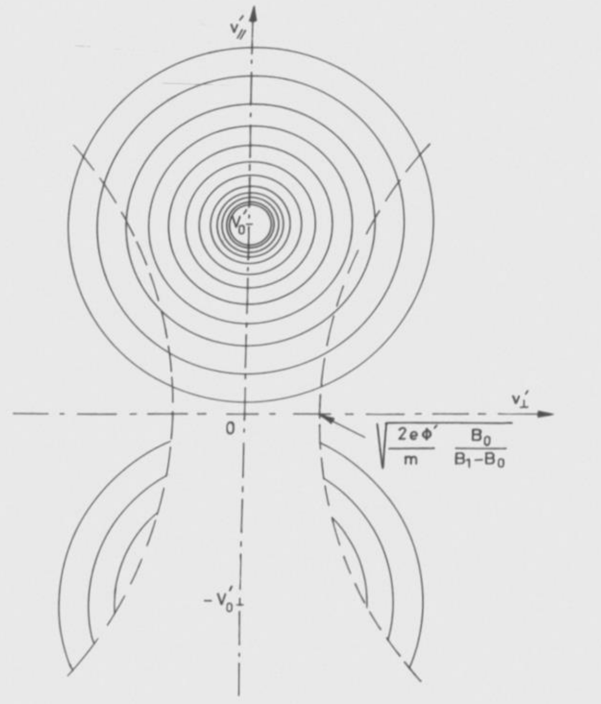

Figure 2.

Schematic representation of isodensity contours of the incoming distribution ($v'_{\|} \geqslant 0$) and reflected distribution ($v'_{\|} \leqslant 0$) in ($v'_{\|}$, $v'_{\perp}$) velocity space. The dashed lines are the separatrix given by eq. (11).

$$\frac{1}{2} m v_{\|}'^2 = \frac{1}{2} m v_{\|}'^2(x) + \Psi(x) \quad (9)$$

with

$$\Psi(x) = \mu(B(x) - B_0) - e(\phi'(x) - \phi'(0)) . \quad (10)$$

Consider incident electrons whose magnetic moment μ is large enough for the pseudo-potential $\Psi(x)$ to have positive values for $x \geqslant 0$. If, as we assume, the functional forms of $(B(x) - B_0)$ and $(\phi'(x) - \phi'(0))$ are alike, then the maximum value Ψ_{max} of $\Psi(x)$ occurs at $x = L$, where both electric potential and magnitude of the magnetic field reach their maximum value. Thus to a good approximation $\Psi_{max} = \mu(B_1 - B_0) - e\phi'$, where we have set $B_1 = B(L)$, $\phi' = \phi'(L)$, and $\phi'(0) = 0$. Then it is seen that any incoming electron having a velocity $v'_{\|}$ in the HT frame such that $\frac{1}{2} m v_{\|}'^2 \leqslant \Psi_{max}$ will not be able to overcome the pseudo potential barrier. In other words, any incoming electron whose magnetic moment and parallel velocity are such that

$$\mu \geqslant \frac{1}{B_1 - B_0}\left(e\phi' + \frac{1}{2} m v_{\|}'^2\right) \quad (11)$$

must be reflected off the shock front.

c) Reflected electrons distribution function

Let $f_{SW}(v'_{\|}, \mu)$ denote the (solar wind) incoming electrons distribution function expressed as a function of velocity variables in the HT frame. Since the distribution function remains constant along phase space trajectories, the reflected electrons distribution function $f_r(v'_{\|}, \mu)$ is simply given by

$$f_r(-v'_{\|}, \mu) = f_{SW}(v'_{\|}, \mu) \quad (12)$$

if $v'_{\|} \geqslant 0$ and $v'_{\|}$, μ satisfy equation (11), $f_r(v'_{\|}, \mu) = 0$ otherwise.

In figure 2 are represented schematically in velocity space ($v'_{\|}$, $v'_{\perp}$) the isodensity contours of the total (incoming + reflected) distribution function, together with the separatrix given by (11). The $v'_{\|} \geqslant 0$ ($v'_{\|} \leqslant 0$) part of the figure describes the incoming (reflected) distribution, respectively. Figure 2 shows that the reflected electrons distribution has a loss-cone character, which is the expected signature of magnetic mirroring effects. The presence of the potential introduces a widening of the loss-cone, particularly at low $v'_{\|}$; as in general the quantity $e\phi'/T_e$ is of the order of 1 (Axford, 1981), this implies that only electrons with suprathermal perpendicular velocities can possibly be reflected (fig. 2). Note that a representation in ($v_{\|}$, $v_{\perp}$) velocity space of the electron distribution function can easily be obtained by operating a translation of figure 2 along the $v'_{\|}$ axis of magnitude $\mathbf{V}_{HT} \cdot \mathbf{b}$ (see equations (4)-(5)).

In order to proceed, a specific form of $f_{SW}(v'_{\|}, \mu)$ has to be adopted. We choose, for the sake of simplicity and easy analytical reduction, the solar wind electrons distribution to be represented by an isotropic drifting Maxwellian

$$f_{SW}(v'_{\|}, \mu) = n_e\left(\frac{m}{2\pi T_e}\right)^{3/2} \times \\ \times \exp\left[-\frac{m}{2T_e}(v'_{\|} - V'_0)^2 - \frac{\mu B_0}{T_e}\right] \quad (13)$$

where n_e and T_e are the upstream electrons density and temperature. The choice (13) gives a rather accurate representation of the cold electrons contribution to the actual solar wind distribution, but has the drawback of not taking into account the solar wind halo population (Feldman *et al.*, 1975). Using equation (13), we readily find that the reflected electrons distribution function integrated over $v'_{\perp}$ (or μ) in the HT frame is simply given by the truncated Maxwellian (truncated since its $v'_{\|} \geqslant 0$ part is excluded) :

$$F_r(v'_{\|} ; v'_{\|} \leqslant 0) = n_e\left(\frac{m}{2\pi T_e}\right)^{1/2} \times \\ \times \exp\left(-\frac{B_0}{B_1 - B_0}\frac{e\phi'}{T_e} - \frac{m}{2T_e}\frac{B_0}{B_1}V_0'^2\right) \\ \times \exp\left[-\frac{m}{2T_e}\frac{B_1}{B_1 - B_0}\left(v'_{\|} + V'_0\left(\frac{B_1 - B_0}{B_1}\right)\right)^2\right] \quad (14)$$

In order to evaluate various moments of $F_r(v'_{\|})$, we may restrict ourselves, again for simplicity, to angles θ_{Bn} large enough so that $V'_0 = V_0 \cos\theta_{Vn}/\cos\theta_{Bn}$ be larger than v_{th} and consequently that the truncation of the Maxwellian (14) be considered as unimportant. When this restriction can be applied, namely when

$$\cos\theta_{Bn} \leqslant V_0 \cos\theta_{Vn}/(2\,T_e/m)^{1/2}\,, \qquad (15)$$

one immediatly finds from equation (14) the reflected electrons mean velocity and temperature in the parallel direction

$$V'_{r\parallel} = -\,V'_0\left(1 - \frac{B_0}{B_1}\right) \qquad (16)$$

$$T_{r\parallel} = T_e\left(1 - \frac{B_0}{B_1}\right). \qquad (17)$$

Coming back to the observer's frame (equations (4) and (6)), the mean velocity of these electrons along the magnetic field lines becomes

$$V_{r\parallel} = -\,V_0\left(2\,\frac{\cos\theta_{Vn}}{\cos\theta_{Bn}}\left(1 - \frac{B_0}{2\,B_1}\right) - \cos\theta_{BV}\right). \qquad (18)$$

The density and perpendicular temperature of the reflected electrons are approximately

$$n_r \simeq n_e\left(1 - \frac{B_0}{B_1}\right)^{1/2} \exp\left(-\,\frac{e\phi'}{T_e}\,\frac{B_0}{B_1 - B_0} - \frac{V_0^2\cos^2\theta_{Vn}}{\cos^2\theta_{Bn}}\,\frac{m}{2\,T_e}\,\frac{B_0}{B_1}\right) \qquad (19)$$

$$T_{r\perp} \simeq T_e\left(1 + \frac{1}{2}\,\frac{B_0}{B_1}\right) + \frac{1}{2}\,\frac{mV_0^2\cos^2\theta_{Vn}}{\cos^2\theta_{Bn}}\,\frac{B_0}{B_1}\left(1 - \frac{B_0}{B_1}\right) + \frac{B_0}{B_1 - B_0}\,e\phi'\,. \qquad (20)$$

The last relation is obtained from

$$n_r\,T_{r\perp} = \int f_r\,\frac{mv_\perp^2}{2}\,\mathrm{d}^3v\,,$$

where f_r is given by equation (12). Note that the reflected electrons density goes to 0 as B_1 tends to B_0, as it should. Also, recall that $T_{r\perp}$ is only a second order moment and bears very little the classical meaning of temperature, since the reflected electrons distribution function has a loss-cone rather than a Maxwellian character in its μ dependence (fig. 2). A particularly interesting and simple result that can be extracted from equations (17) and (18) is that for angles θ_{Bn} close enough to 90°, the average kinetic energy per reflected electron $E_{r\parallel} = 1/2(mV_{r\parallel}^2 + T_{r\parallel})$, as measured in the observer's frame along the field lines, is to leading order in $1/\cos\theta_{Bn}$

$$E_{r\parallel} \sim 4\,\frac{\cos^2\theta_{Vn}}{\cos^2\theta_{Bn}}\,\frac{1}{2}\,mV_0^2\,, \qquad (21)$$

having neglected $(B_0/2\,B_1)$ in front of 1.

Thus, the reflected electrons distribution function (12) and, for a particular choice of f_{SW}, its macroscopic moments equations (17)-(20) as expressed in the observer's frame, describe completely the properties of the electrons reflected by magnetic mirroring at the shock surface and flowing back upstream along the field lines, at least when the shock is nearly perpendicular (inequality (15)). We see that $V_{r\parallel}$ and $E_{r\parallel}$ depend very weakly on the reflection mechanism and the shock structure through the usually small ($\sim 1/4$) correcting factor B_0/B_1, and quite sensitively on the geometry under consideration (through the $1/\cos\theta_{Bn}$ factor) ; whereas n_r and $T_{r\perp}$ depend sensitively on both the shock structure (ϕ', B_0/B_1) and geometrical angles.

Other quantities of interest for subsequent comparison with observations are the flux of reflected electrons along the field $J_{r\parallel} = n_r\,V_{r\parallel}$, the temperature anisotropy $A_r = T_{r\perp}/T_{r\parallel}$, and the average pitch-angles defined by $\tan^2\xi_r = \langle v_\perp^2\rangle/\langle v_\parallel^2\rangle$, where the sign $\langle\ \rangle$ represents an average over the reflected electrons distribution ; the latter can be rewritten in the form

$$\tan^2\xi_r = \frac{2\,A_r}{1 + \dfrac{mV_{r\parallel}^2}{T_{r\parallel}}}\,. \qquad (22)$$

An inspection of equations (17), (18), (20) and (22) shows that as θ_{Bn} tends to 90°, the average pitch-angle ξ_r does not tend to zero, but to a limit whose value is

$$\tan^{-1}\left[\frac{1}{2}\left(\frac{B_0}{B_1}\right)^{1/2}\left(1 - \frac{B_0}{B_1}\right)^{1/2}\Big/\left(1 - \frac{B_0}{2\,B_1}\right)\right] \simeq$$

$$\simeq \tan^{-1}\left(\frac{1}{2}\left(\frac{B_0}{B_1}\right)^{1/2}\right).$$

III. DISCUSSION

We evaluate in this section a few orders of magnitude of the density, energy and other characteristics of the reflected electrons as given by our model. In order to be able to relate the results to available observations, we choose a series of parameters which we consider as fairly typical of the nearly perpendicular Earth's bow shock, namely $M_A = 6$, $\beta_e = 1$, $\theta_{Vn} = 30°$, $\theta_{BV} = 45°$, $B_1/B_0 = 4$, and $e\phi'/T_e = 5$. Here M_A represents the Alfvén Mach number,

$$M_A = V_0\cos\theta_{Vn}(4\,\pi n_e\,M)^{1/2}/B_0\,.$$

The results are summarized in tables 1 and 2. We calculate in table 1 only dimensionless quantities, the density n_r normalized to n_e, the flux $J_{r\parallel}$ normalized

Table 1

Density (n_r) normalized to n_e, flux ($J_{r\parallel}$) normalized to $n_e V_0$, energy/charge ($E_{r\parallel}$) normalized to T_e, temperature anisotropy (A_r) and average pitch-angle (ξ_r) of reflected electrons in observer's frame, as a function of θ_{Bn}, for $M_A = 6$, $\beta_e = 1$, $e\phi'/T_e = 5$, $\theta_{BV} = 45°$, $\theta_{Vn} = 30°$, and $B_1/B_0 = 4$.

θ_{Bn}	n_r/n_e	$J_{r\parallel}/n_e V_0$	$E_{r\parallel}/T_e$	A_r	ξ_r
83°	$1.1.10^{-1}$	1.3	4.1	4.0	41°
86°	$6.0.10^{-2}$	1.2	12	4.7	28°
88°	$2.9.10^{-3}$	$1.2.10^{-1}$	49	7.7	19°
88.5°	$1.3.10^{-4}$	$7.2.10^{-3}$	86	11	17°
89°	$1.7.10^{-8}$	$1.4.10^{-6}$	200	20	15°
89.5°	10^{-29}	10^{-27}	790	68	14°

Table 2

Energy/charge ($E_{r\parallel}$) and flux ($J_{r\parallel}$) versus θ_{Bn} for same parameters as in table 1, with in addition $V_0 = 400$ km/s and $n_e = 10$ cm^{-3}.

θ_{Bn}	$E_{r\parallel}$	$J_{r\parallel}$ (cm^{-2} s^{-1})
83°	72 eV	$5.2.10^8$
86°	210 eV	$4.8.10^8$
88°	850 eV	$4.8.10^7$
88.5°	1.5 keV	$2.9.10^6$
89°	3.4 keV	$5.2.10^2$
89.5°	14 keV	10^{-19}

to $n_e V_0$, the average kinetic energy per charge $E_{r\parallel}$ along the field lines normalized to T_e, the temperature anisotropy A_r and the average pitch-angle ξ_r of the reflected electrons, in the observer's frame, for various values of θ_{Bn} satisfying equation (15). In table 2 are listed the dimensioned quantities $E_{r\parallel}$ (in keV) and the flux $J_{r\parallel}$ (in cm^{-2} s^{-1}), calculated using the additional data $n_e = 10$ cm^{-3} and $V_0 = 400$ km/s, also as a function of θ_{Bn}.

Table 1 (and also fig. 3) shows that the average energy per charge increases from suprathermal ($E_{r\parallel}/T_e \sim 4$) to energetic ($E_{r\parallel}/T_e \sim 200$) while the normalized density decreases from 10^{-1} to $\sim 10^{-8}$ as θ_{Bn} increases from 83° to 89°. The normalized flux also decreases,

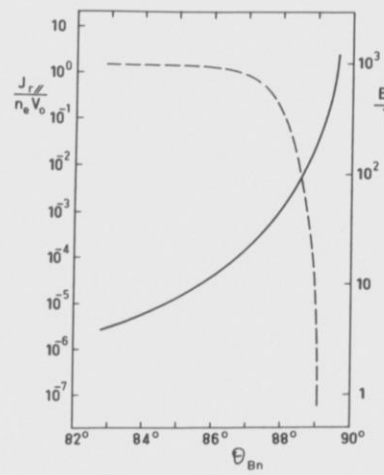

Figure 3.

Normalized energy per charge (solid line) and normalized flux (dashed line) as a function of θ_{Bn}.

although somewhat less than the density (since the flux is the product of the density by an increasing function of θ_{Bn}, see equation (18)), from 1.3 when $\theta_{Bn} = 83°$ to $\sim 10^{-6}$ when $\theta_{Bn} = 89°$ (table 1 ; fig. 3). In addition, table 1 shows that the temperature anisotropy is large compared to 1 and increases with θ_{Bn} ; however, due to the fact that $E_{r\parallel}$ increases faster than A_r as a function of θ_{Bn}, the average pitch-angle decreases (equation (22)), from about 40° at $\theta_{Bn} = 83°$ to 15° at $\theta_{Bn} = 89°$. Turning now to dimensioned quantities, it is seen (table 2) that energetic electrons in the range ~ 100 eV to ~ 5 keV are readily obtained with significant fluxes, from 5 10^8 cm^{-2} s^{-1} at low energies (100 eV) to 10^2 cm^{-2} s^{-1} at 4 keV. As θ_{Bn} increases beyond 89°, the energy can reach very high values (eq. (21)) (up to 50 keV when $\theta_{Bn} = 89.7°$) but the flux drops drastically down to negligible values as soon as the energy exceeds 5 keV (table 2).

Before discussing these results in the light of the observations, it is worth pointing out that the reflected electrons distribution function as predicted by our model describes the reflected electrons very near (but upstream of) the shock front and consequently depends only on local shock parameters, whereas the distribution as measured farther upstream of the shock by the spacecraft is made of electrons coming from different parts of the curved bow shock (non local distribution). This is due to the fact, first noted by Asbridge *et al.* (1968), that the convection of the field lines by the solar wind enforces the backstreaming electrons to move along a direction which makes an angle δ with **b**, given by $\tan \delta = V_0 \sin \theta_{BV}/v_\parallel$. The higher the energy of a backstreaming electron, the smaller the angle δ (δ is of the order of a degree for 1 keV electrons). Thus, the spacecraft at a given position in space will detect fast electrons originating at the bow shock surface on the field line which connects the spacecraft to the bow shock, as well as slower electrons emitted at other portions of the bow shock and subsequently convected back to the spacecraft. We expect these ballistic effects to be small when electrons are sufficiently energetic ($\geqslant 1$ keV) or when the observing spacecraft is close enough to the bow shock surface, say, less than a few Earth radii.

Keeping this limitation in mind, the comparison between observational data and the present model leads to the following remarks :

1) High energy electrons ($\geqslant 2$ keV) come from a very small region of the bow shock corresponding to less than a degree in angle θ_{Bn} near the point of tangency of $\mathbf{B}_0$ with the bow shock surface ($\theta_{Bn} = 90°$) ; lower energy electrons (~ 100 eV-2 keV) come from a broader region of the bow shock (6° in angle θ_{Bn} ; table 2) and the energy of the reflected electrons decreases with decreasing θ_{Bn} (fig. 3). These features of the theory are fully consistent with the experimental findings of Anderson *et al.* (1979) and Feldman *et al.* (1983).

2) The quantity of reflected electrons as predicted by the theory is in reasonable agreement with observational data. The flux of ~ 2 keV electrons is measured to be $\sim 10^4$ cm^{-2} s^{-1} ster^{-1} keV^{-1} (Anderson, 1981 ; Parks *et al.*, 1981 ; Anderson *et al.*, 1981). Taking as an

order of magnitude a solid angle containing the directions of energetic electrons to be about 0.1 π, and a bandwidth of the detector to be 0.3 keV, we find that the measured flux is of the order of 10^3 cm^{-2} s^{-1}, which compares reasonably with the flux at 2 keV found in the theory, $\sim 3\ 10^5$ cm^{-2} s^{-1} (table 2) (we would have considered as quite satisfactory an agreement within a factor 100 for the density or flux of the energetic electrons, given the strong sensitivity of the reflected electrons density to local shock parameters [eq. (23), table 2]). We can also estimate the extra heat flux induced by the energetic electrons at, say, 1 keV by $Q \simeq \frac{1}{2} mn_r(V_{r\parallel} - V_0 \cos\theta_{BV})^3 \sim 6\ 10^{-2}$ ergs cm^{-2} s^{-1} with the same parameters as previously, which compares also rather well with measured values $\sim 10^{-2}$ ergs cm^{-2} s^{-1} (Ogilvie *et al.*, 1971).

3) It is found in Section II that the average pitch-angle of the energetic electrons tends to increase with decreasing energies (table 1). This trend is consistent with the observations by Anderson (1981) that the pitch-angle distribution has the tendency to broaden at low energies.

4) It has been observed by Feldman *et al.* (1982, 1983) that at low energies ($\sim$ 50 eV), pitch-angle scans through the electrons distribution function at constant energy in a two-dimensional velocity space show two peaks which bracket the direction of the ambient field $\mathbf{B}_0$. This may well be the signature in a two-dimensional velocity space of the loss-cone distribution function of the reflected electrons predicted by the present theory (see also the Section 5 of Feldman *et al.*, 1983).

5) Table 2 shows that the flux of energetic electrons is a monotonically decreasing function of the energy per charge, in agreement with observations (Reasoner, 1975 ; Anderson, 1981 ; Parks *et al.*, 1981). However, the shape of the curve $J_{r\parallel}(E_{r\parallel})$ as found in the theory differs substantially at high energies ($\gtrsim 5$ keV) from that inferred by experimenters (Anderson, 1981 ; Parks *et al.*, 1981). Inspection of equations (18), (19) and (21) shows that the flux $J_{r\parallel}$ varies with $E_{r\parallel}$ roughly as $E_{r\parallel}^{1/2} \exp(-pE_{r\parallel})$, where p is a constant depending on local shock parameters (but not on θ_{Bn}) ; this exponential dependence makes the flux to be too large at low energies ($\leqslant$ 500 eV) and to drop drastically down to negligible values for $E_{r\parallel} \geqslant 5$ keV (table 2). On the other hand, experimental evidence (Anderson, 1981 ; Parks *et al.*, 1981 ; Potter, 1981) shows that the function $J_{r\parallel}(E_{r\parallel})$ is best approximated by a power law function, which results in a much smoother decrease of $J_{r\parallel}$; negligible values of the flux are obtained at higher energies, $E_{r\parallel} \simeq 50$ keV. We attribute this discrepancy to the fact that we have chosen the solar wind electron distribution function to be represented simply by a single Maxwellian. Clearly, since the basic mechanism of energization considered here is a simple mirror reflection of electrons in an appropriate reference frame, the features of the reflected electrons distribution function bear much similarity with those of the original incoming distribution function (see equation (12)) ; hence the nearly exponential dependence of the flux as a function of energy. We note that the high energy reflected electrons ($\geqslant$ 5 keV) come primarily from the tail of the solar wind distribution, which in reality consists of a halo that can be modelled by a series of modified Lorentzian functions (Feldman *et al.*, 1982). The inclusion in the theory of a detailed description of the solar wind distribution function is very likely to result in a much smoother decrease of the flux as a function of energy, in closer agreement with observations.

IV. CONCLUSIONS

We have presented a theory of electron energization by nearly perpendicular bow shock waves based essentially on adiabatic mirror reflection of the incoming electrons by the jump in the magnetic field arising in the shock transition region. The theory, as applied to the Earth's bow shock, shows satisfactory agreement with observational data, qualitatively and often quantitatively. In the light of the theory, nearly perpendicular bow shock waves thus appear as natural energizers of the solar wind electron plasma. The energy range of energetic electrons produced in a direct manner by bow shocks is expected, however, to have an upper limit of a few tens of keV. Indeed, very high energy ($\geqslant$ 220 keV) electron bursts observed in the upstream region of the Earth's bow shock moving sunward appear to have their origin inside the magnetosphere rather than at the bow shock surface (Krimigis *et al.*, 1978). An important feature of the theory is that electron energization should occur regardless of the fact that the shock is supercritical or not. Most available data are concerned with the Earth's bow shock, which in general is supercritical ; there exists also indications that some of the interplanetary shocks (usually subcritical) are indeed able to reflect and energize incoming electrons (Potter, 1981), but whether or not these shocks are in fact nearly perpendicular has yet to be confirmed observationally. Finally, we point out that the domain of applicability of the theory may not be restricted to shock waves in the interplanetary space. In the first place, enhanced radiation at the electron plasma frequency and its first harmonic has been detected in the vicinity of an essentially perpendicular shock wave produced in theta-pinch laboratory experiments (Chin-Fatt and Griem, 1970), whose origin is very likely to be correlated to the emission of energetic electrons by the shock wave. In the second place, there is now good experimental evidence that the solar type II bursts containing herringbone structure in their dynamic radio spectra originate from shock waves propagating more or less perpendicular to open magnetic fields in the solar corona (Stewart and Magun, 1980). It has been suggested (Holman and Pesses, 1983) that the mechanism of electron energization described in the present work is operable for producing the herringbone structure of such solar type II bursts. We feel that the knowledge of the reflected electrons properties (density, mean velocity, temperature anisotropy, etc.) such as derived in this paper may be very helpful for further analysing the radiation emission mechanism.

Acknowledgements

Helpful discussions with J. D. Scudder, J. L. Steinberg, C. Lacombe and W. C. Feldman are gratefully acknowledged.

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