Useful Asymptotic Properties

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Some useful properties:

- $O(1) \subset O(\log n) \subset O(n) \subset O(n \log n) \subset O(n^2) \subset O(2^n) \subset O(3^n) \subset O(n^n)$
- All logarithms are $\Theta(\log n)$ by change of base rule (differ by only a constant factor).
- Asymptotic notation only cares about "highest-growing" terms. For example, $n^2 + n = \Theta(n^2)$.
- Asymptotic notation does not care about leading constants.
- Any exponential with base > 1 grows faster than any polynomial. For example, $n^{100} = O(1.01^n)$, but $n^{100} \neq \Omega(1.01^n)$.
- The base of the exponential matters. For example, $3^n = O(4^n)$, but $3^n \neq \Omega(4^n)$.
- If $f_1 = O(g_1(n)), f_2 = O(g_2(n))$, then
  - $f_1 f_2 = O(g_1 g_2)$.
  - $f_1 + f_2 = O(\max\{g_1, g_2\})$.